Stat 403 (3) Stochastic Processes Random walks, Markov chains, Poisson processes, continuous time Markov chains, birth and death processes, exponential models, and applications of Markov chains. [3-0-0] Prerequisite: STAT 303. Learning Outcomes: After completing this course, you will be able to: Understand and describe stochastic processes mathematically as both 1. a collection of time-indexed random variables on a common sample space, and 2. a random trajectory, or sample path, through a given state space. Understand the notion of an i.i.d. (independent and identically distributed) sequence of random variables. Study certain processes associated to i.i.d. sequences, such as record values for non-negative random variables, and runs of 1s for Bernoulli random variables. Construct a random walk from an i.i.d. sequence of real-valued random variables. Define Markov chains in discrete time using transition probabilities, and in continuous time using transition rates. Construct a Markov chain model, based on a given description. Verify the Markov property for a given process. Compute the probability distribution for the location of a Markov chain at a given fixed time, using the Chapman-Kolmogorov (discrete time) or forward/ backward Kolmogorov (continuous-time) equations. Define stopping times and understand the strong Markov property. Determine the communicating classes of a Markov chain from its transition probabilities/rates and use these to describe its long-term behaviour. Understand the phenomenon of periodicity in a discrete time Markov chain. Use conditioning and first-step analysis to obtain and solve sets of equations describing important objects associated to Markov chains, such as the hitting probability and expected hitting time of a set, and stationary distributions, when they exist. Understand the notions of recurrence and transience of a stochastic process. Determine the recurrence or transience of various random walks. Construct and analyze various models in discrete and/or continuous time and study their characteristic properties, such as: branching processes and their survival probability and asymptotic rate of growth, 2. the radioactive decay model, and the time until complete decay, 3. the ehrenfest urn model and its stationary distribution, 4. queuing models, existence or non-existence of a stationary distribution and expected wait times, 5. general birth and death processes, probability of divergence, hitting times and stationary distributions, 6. the basic insurance model and the ruin probability, 7. the Moran model of population genetics, and its probability of fixation and expected time until fixation, 8. the stochastic SIR model of infection spread in a finite population, and its final size. Understand the connection between discrete time and continuous time Markov chains obtained by the embedded Markov chain. Construct a Markov chain mathematically: 1. from an i.i.d. collection of uniform random variables, in discrete time, and 2. from an i.i.d. collection of exponential random variables, and an i.i.d. collection of uniform random variables, in continuous time. • Use the technique of coupling to study the convergence of a Markov chain to its stationary distribution, when one exists. Construct and analyze the Poisson point process and the corresponding counting process, the Poisson process. Understand its relationship with exponential random variables and with i.i.d. uniform random variables.